

The Undecided-State Dynamics

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Abstract

We formalize the basic properties of the synchronous undecided-state dynamics with two opinions on the complete graph: the exact one-round expectations of the three counts, the resulting growth of the bias by the factor $1 + q/n$ (the undecided nodes amplify the current majority), and almost-sure absorption in a monochromatic configuration.

1 The model

Definition 1 (Undecided-state dynamics). Each of n nodes holds opinion a , opinion b , or is undecided. In a round every node samples a node uniformly at random (with replacement, possibly itself): an undecided node adopts the sampled opinion, and a decided node that samples the other opinion becomes undecided. We write a , b , q for the numbers of a -, b - and undecided nodes.

2 One round in expectation

Lemma 2 (Per-node probabilities). *The state of a node after a round depends only on the node it samples, which is uniform; so an a -node stays a with probability $1 - b/n$ and an undecided node becomes a with probability a/n .*

Proof. The average over all sampling functions of a function of one coordinate is the uniform average. $\square$

Theorem 3 (Expected counts and bias). *After one round, the expected numbers of a -, b - and undecided nodes are $a(n - b + q)/n$, $b(n - a + q)/n$ and $(q^2 + 2ab)/n$; hence the expected bias is $(a - b)(1 + q/n)$.*

Proof. Sum Lemma 2 over the nodes; $a(n - b + q) - b(n - a + q) = (a - b)(n + q)$. $\square$

3 Absorption

Theorem 4 (Almost-sure absorption). *Monochromatic configurations are fixed points, and from every configuration the probability of not being monochromatic after t rounds tends to 0.*

Proof. If some node w is decided, the round in which everybody samples w , repeated twice, makes the configuration monochromatic; it has positive probability. Conclude with the shared absorption theorem. $\square$

4 The sequential dynamics: approximate majority

Definition 5 (Sequential undecided-state dynamics). At each step an ordered pair (initiator, responder) of distinct agents is drawn uniformly among the $n(n-1)$ such pairs, independently of the past; only the responder updates, by the rule of Definition 1 applied to the initiator's state. This is the approximate-majority protocol of Angluin, Aspnes and Eisenstat (Distributed Computing 21, 2008), with x, y and blank written a, b and u .

Lemma 6 (One-step formula). *The expectation of a function of the next configuration that depends only on the states of the initiator and the responder is its value on idle interactions plus the four state-changing transitions xb, yb, xy, yx , weighted by aq, bq, ab, ab pairs out of $n(n-1)$ (with q the number of blank agents).*

Lemma 7 (Weighted supermartingales). *If one step never increases $e^\varphi F$ in expectation, then $e^{\sum_{\text{path}} \varphi} F$ has expectation at most F at every finite time, and Markov's inequality bounds the probability that the path sum of φ is large.*

Lemma 8 (Six potentials). *The potentials $1/((a-b)^2 + 4n)$ (state-changing interactions), 1 (central region), $1/(a+b)$ (blank corner), $3b+q+1$ and $3a+q+1$ (the two opinion corners), and $e^{-\lambda \max(a-b, 0)}$ (the gap stays positive) satisfy the hypothesis of Lemma 7 with suitable weights.*

Theorem 9 (Convergence; AAE Theorem 1). *For every c there is C such that from every non-blank configuration of $n \geq 2$ agents, after $T \geq Cn \log n$ interactions all agents agree, with probability at least $1 - C/n^c$.*

Theorem 10 (Correctness; AAE Theorem 2). *For every c there is C such that if initially $a - b \geq C\sqrt{n} \log n$, then after $T \geq Cn \log n$ interactions all agents hold a , with probability at least $1 - C/n^c$.*